[1] L. Landau, "On the vibration of the electric plasma," J. Phys. (USSR) 10, 25–34 (1946).
[2] A. Vlasov, "On the kinetic theory of an assembly of particles with collective interaction," J. Phys. (USSR) 9, 25–40 (1945).
[3] A. W. Chao, *Physics of Collective Beam Instabilities in High Energy Accelerators* (Wiley, New York, 1993), pp. 218–268.
[4] B. Fak, K. Guckelsberger, M. Korfer, R. Scherm, and A. J. Dianoux, "Elementary excitations in superfluid ^{3}He–^{4}He mixtures: Pressure and temperature dependence," Phys. Rev. B 41, 8732–8748 (1990).
[5] R. Baier, H. Nakkagawa, A. Niegawa, and K. Redlich, "Production rate of hard thermal photons and screening of quark mass singularity," Z. Phys. C 53, 433–438 (1992).
[6] S. Strogatz, R. Mirollo, and P. Matthews, "Coupled nonlinear oscillators below the synchronization threshold: Relaxation be generalized Landau damping," Phys. Rev. Lett. 68, 2730–2733 (1992).
[7] J. Buck, "Synchronous rhythmic flashing of fireflies. II," Q. Rev. Biol. 63, 256–289 (1988).
[8] A. T. Winfree, *The Geometry of Biological Time* (Springer, New York, 1980), pp. 315–336.
[9] K. M. Case, "Plasma oscillations," Ann. Phys. 7, 349–364 (1959).
[10] N. G. Van Kampen, "On the theory of stationary waves in plasmas," Physica 21, 949–963 (1955).
[11] J. H. Malmberg and C. B. Wharton, "Collisionless damping of electrostatic plasma waves," Phys. Rev. Lett. 13, 184–186 (1964).
[12] P. C. Clemmow and J. P. Dougherty, *Electrodynamics of Particles and Plasmas* (Addison-Wesley, Reading, MA, 1969), pp. 251–264.
[13] C. Heng-jin and C. Pei-ren, "Exact solution to linear Vlasov equation and discussions on Landau damping, instabilities and resonant interactions," Sci. China A 34, 324–335 (1991).
[14] Dwight Nicholson, *Introduction to Plasma Theory* (Wiley, New York, 1983), pp. 70–124.
[15] The reader may be wondering why a simpler uniform distribution with $\rho(\omega)=1/(2\Delta\omega)$ for $|\omega-\omega_0|<\Delta\omega$ was not used. The reason, as shown in Appendix B, is that the unphysical discontinuities at $\omega=\omega_0\pm\Delta\omega$ gives an 'anomalous' stability diagram.
[16] H. M. Smith, "Synchronous flashing of fireflies," Science 82, 151–152 (1935).
[17] O. Penrose, "Electrostatic instabilities of a uniform non-Maxwellian plasma," Phys. Fluids 3, 258–265 (1960).
[18] G. Backus, "Linearized plasma oscillations in arbitrary electron velocity distributions," J. Math. Phys. 1, 178–191 (1960).
[19] J. Dawson, "On Landau damping," Phys. Fluids 4, 869–874 (1961).
[20] M. Abramowitz and I. Stegun, *Handbook of Mathematical Functions* (Dover, New York, 1972), pp. 228–251.
[21] J. Li, and G. O. Spies, "Initial value problem for plasma oscillations," Phys. Fluids B 3, 1158–1166 (1991).

Velocity fields inside a conducting sphere near a slowly moving charge

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Explicit expressions for first-order electric and magnetic fields created inside a conducting sphere by a nearby slowly moving charge are given. They are found to be independent of the sphere radius. On the contrary, outer first-order fields, which are also computed, depend on it. The energy dissipation by Joule effect is calculated and shown to agree with the external first-order work done on the charge to maintain its uniform motion.

I. INTRODUCTION

In most textbooks on electromagnetism the well-known low penetration of electromagnetic waves inside conductors is discussed. However, no attention is usually paid to the penetration of electromagnetic velocity fields, perhaps because in most cases the charges creating them are far away from the conductors, in such a way that velocity fields are unimportant and only radiation fields matter.

But when charges are moving close to a conductor, they produce a charge redistribution on the surface and currents inside the conductor that react against the moving charges. For instance, to test the weak equivalence principle for antimatter, several experiments involving antiprotons, negative hydrogen ions, positrons, and electrons under the influence of the Earth's gravitational field are in progress in different laboratories.[1] They are, in general, drift-tube experiments in which a burst of particles and antiparticles are emitted upwards or downwards and detected at the end to show differences between particles and antiparticles under the action of gravity. During the flight, particles are affected by residual gas, radiation, gravity, driving electric and magnetic fields, and a frictional force due to the image current in the nearby conductors. This image current is not a surface effect but rather it is equivalent to a current density distribution inside conductors that dissipates energy by the Joule effect. This has to be taken into consideration to properly interpret the experimental results.

In another context, Boyer[2] has suggested that the Aharonov–Bohm effect implies the existence of classical electromagnetic forces between charged particles and solenoids. These forces are a consequence of the electric and magnetic velocity fields.

On the other hand dynamical methods to measure the induced charge on conductors[3] are based upon the measurement of the surface charge distribution produced by a very close passage of charged projectiles. This surface charge distribution, produced by internal currents induced by the velocity fields, is not a skin-depth effect but an effect on the bulk of the conductor.

Thus, it seems interesting to pay attention to the penetration of velocity fields into conductors. In this context, Boyer[4] has studied the first-order penetration of the fields produced by a charge moving parallel to a conducting in-

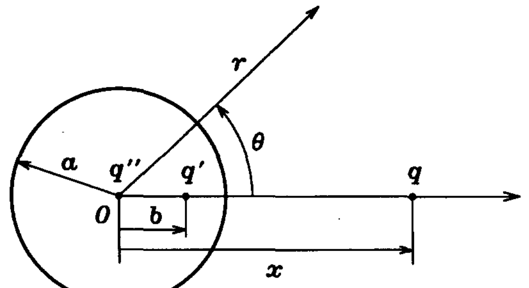

Fig. 1. Charge and image charges of the outer equivalent electrostatic system.

finite wall filling a half-space. The shielding of the magnetic field of a charge moving near a conducting surface has been discussed by Furry.[5] The relative importance of magnetic skin effects in the case of a line charge moving with arbitrary velocity parallel to an infinite plane conductor has been analyzed by Jones.[6]

The aim of the present paper is to study a more realistic situation in which all relevant quantities can still be computed in explicit form. We shall consider the electric and magnetic fields induced inside a conducting sphere by a point charge moving at constant velocity in the radial direction.

In Sec. II we consider the static situation corresponding to the lowest order contribution. Electric field and potential are calculated in Sec. III to first order in the charge velocity. Similarly, interior and exterior magnetic fields are computed to the same order in Sec. IV while Sec. V is devoted to the calculation of the energy dissipated inside the sphere by the Joule effect and to check that the power developed by the external force on the particle compensates that energy loss. Finally, Sec. VI contains some comments on the validity of our approach and on the functional dependence of inner and outer fields on the sphere radius and on the resistivity.

II. THE LOWEST ORDER FIELDS

Let us consider a point charge q located at a distance x from the center of an uncharged conducting sphere of radius a, resistivity η, electric permittivity ϵ_0, and magnetic permeability μ_0 (see Fig. 1). The charge is moving along the radial direction with constant velocity $v=dx/dt$. We shall assume that v is small and proceed to analyze the problem with the perturbative approach described in Boyer's paper.[4] Every quantity (fields, potentials, charge, and current densities) will be expanded in powers of v/c:

$$f=\sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{v}{c}\right)^n f_n=\sum_{n=0}^{\infty}f^{(n)},\quad f^{(n)}\equiv\frac{1}{n!}\left(\frac{v}{c}\right)^n f_n,$$

f_n being the nth derivative with respect to v/c evaluated at $v/c=0$. We shall retain only up to terms of first order in v/c, because as discussed in Sec. VI we expect them to give the only non-negligible contributions.

The lowest order fields correspond to the well-known static case in which the charge is at rest ($v=0$). There is no magnetic field, $B^{(0)}=0$, and the electric field $E^{(0)}$ vanishes inside the conductor, the sphere being equipotential. Outside the sphere, $E^{(0)}$ is equal to the sum of the electrostatic fields due to charge q located at point x and two image charges $q'=-qa/x$ and $q''=qa/x$ placed at a distance $b=a^2/x$ from the center and at center, respectively[7] (see Fig. 1). Furthermore, the surface charge density on the sphere is obtained from the discontinuity of the electric field and is given by

$$\sigma^{(0)}(\theta)=\frac{q}{4\pi ax}-\frac{q(x^2-a^2)}{4\pi a(x^2-2xa\cos\theta+a^2)^{3/2}}. \tag{1}$$

If the charge is now moving with small velocity, $v=dx/dt$, the surface charge density is changing and there will appear inside the sphere a current density whose radial component on the surface is given by the first-order continuity equation

$$\frac{\partial\sigma^{(0)}}{\partial t}-j_r^{(1)}=v\frac{\partial\sigma^{(0)}}{\partial x}-j_r^{(1)}=0, \tag{2}$$

and thus

$$j_r^{(1)}(a,\theta)=\frac{qv}{4\pi a}\left[\frac{x^3+x^2a\cos\theta-5xa^2+3a^3\cos\theta}{(x^2-2xa\cos\theta+a^2)^{5/2}}-\frac{1}{x^2}\right]. \tag{3}$$

It is worth noting that we have assumed as in Boyer's work[4] that there is no surface current density because, due to the ohmic nature of the conductor, it would imply a singular tangential component of the electric field on the surface that would be incompatible with the electric field matching conditions. It turns out then that the change of the surface charge density is the consequence of a volume effect.

III. FIRST-ORDER ELECTRIC FIELDS

The second term of the electric field

$$\mathbf{E}=-\text{grad}\,\phi-\frac{\partial\mathbf{A}}{\partial t}=-\text{grad}\,\phi-v\frac{\partial\mathbf{A}}{\partial x} \tag{4}$$

is of second order in v/c, because cA is already of first order. As a consequence, the first-order electric field $\mathbf{E}^{(1)}$ will be conservative (curl $\mathbf{E}^{(1)}=0$) and will be expressed in the form $\mathbf{E}^{(1)}=-\text{grad}\,\phi^{(1)}$ in terms of the first-order potential $\phi^{(1)}$ that must satisfy Laplace's equation.

Due to the axial symmetry and the finiteness of potential at the origin, the solution of Laplace's equation inside the sphere has the following form in terms of Legendre polynomials:[8]

$$\phi_{\text{int}}^{(1)}(r,\theta)=\sum_{n=0}^{\infty}A_n r^n P_n(\cos\theta). \tag{5}$$

Thus the radial component of the interior electric field is given by

$$E_{r\text{int}}^{(1)}(r,\theta)=-\frac{\partial\phi_{\text{int}}^{(1)}}{\partial r}=-\sum_{n=1}^{\infty}nA_n r^{n-1}P_n(\cos\theta). \tag{6}$$

Under the assumption that the conducting sphere is ohmic and has resistivity η, we have $E_{r\text{int}}^{(1)}(a,\theta)=\eta j_r^{(1)}(a,\theta)$ on the sphere surface, i.e.,

$$E^{(1)}_{r\mathrm{int}}(a,\theta)=-\sum_{n=1}^{\infty} nA_n a^{n-1}P_n(\cos\theta)$$
$$=\frac{qv\eta}{4\pi a}\left(\frac{x^3+x^2a\cos\theta-5xa^2+3a^3\cos\theta}{(x^2-2xa\cos\theta+a^2)^{5/2}}-\frac{1}{x^2}\right). \tag{7}$$

Using the generating function for Legendre polynomials

$$f(z,u)=\frac{1}{\sqrt{z^2-2uz+1}}=\sum_{n=0}^{\infty}\frac{1}{z^{n+1}}P_n(u), \tag{8}$$

we get

$$2z\frac{\partial^2 f}{\partial z^2}+3\frac{\partial f}{\partial z}=\frac{z^3+z^2u-5z+3u}{(z^2-2zu+1)^{5/2}}$$
$$=\sum_{n=0}^{\infty}\frac{(2n+1)(n+1)}{z^{n+2}}P_n(u). \tag{9}$$

By comparing Eqs. (7) and (9) with $z=x/a$ and $u=\cos\theta$ one can readily obtain

$$A_n=-\frac{qv\eta}{4\pi}\frac{(2n+1)(n+1)}{nx^{n+2}}\quad\text{for } n\geqslant 1. \tag{10}$$

Since the coefficients A_n do not depend on a, we see from Eqs. (6) and (7) that both series have exactly the same mathematical structure, and thus the sum of series (6) can be obtained by merely substituting r for a in Eq. (7):

$$E^{(1)}_{r\mathrm{int}}(r,\theta)=\frac{qv\eta}{4\pi r}\left[\frac{x^3+x^2r\cos\theta-5xr^2+3r^3\cos\theta}{(x^2-2xr\cos\theta+r^2)^{5/2}}-\frac{1}{x^2}\right]. \tag{11}$$

Notice that we only need A_n for $n\geqslant 1$, because the terms between square brackets cancel for $r/x\to 0$. As a consequence, there is no $1/x^2$ term and the field behaves as $1/x^3$ for large x.

The general solution of the partial differential Eq. (6) is

$$\phi^{(1)}_{\mathrm{int}}(r,\theta)=-\int E^{(1)}_{r\mathrm{int}}(r,\theta)dr+g(\theta), \tag{12}$$

where the arbitrary function $g(\theta)$ can be computed by using the condition $\phi^{(1)}_{\mathrm{int}}(0,\theta)=A_0$ which follows from Eq. (5). A_0 could in principle depend on x, but if the sphere is uncharged then $A_0=0$, as we shall see below when computing the exterior potential. Taking this into account, one gets for the potential inside the sphere ($r\leqslant a$):

$$\phi^{(1)}_{\mathrm{int}}(r,\theta)=\frac{qv\eta}{4\pi}\left(\frac{\ln(x^2-xr\cos\theta+xz)-\ln(2x^2)+3}{x^2}-\frac{3x^2-4xr\cos\theta+r^2}{xz^3}\right), \tag{13}$$

where

$$z\equiv\sqrt{x^2-2xr\cos\theta+r^2}. \tag{14}$$

One can check explicitly that this potential satisfies Laplace's equation and the boundary condition for $\partial\phi^{(1)}_{\mathrm{int}}/\partial r$ at $r=0$ given in Eq. (7). From Eqs. (13) and (14) one can easily compute the remaining component $E^{(1)}_{\theta\mathrm{int}}$ of the electric field inside the sphere:

$$E^{(1)}_{\theta\mathrm{int}}(r,\theta)=\frac{qv\eta}{4\pi x^2 r\sin\theta}\times\left(\frac{x^2r(x^2-9r^2)\cos^2\theta+x(5r^4+10r^2x^2+x^4)\cos\theta-r(r^4+r^2x^2+6x^4)}{(x^2-2xr\cos\theta+r^2)^{5/2}}-\cos\theta\right). \tag{15}$$

By using the condition that the potential vanishes at infinity, outside the conducting sphere the electric potential can be expanded in the form[8]

$$\phi^{(1)}_{\mathrm{ext}}(r,\theta)=\sum_{n=0}^{\infty}\frac{B_n}{r^{n+1}}P_n(\cos\theta) \tag{16}$$

and the coefficients B_n can be computed from the first-order continuity condition $\phi^{(1)}_{\mathrm{int}}(a,\theta)=\phi^{(1)}_{\mathrm{ext}}(a,\theta)$, which gives $B_n=a^{2n+1}A_n$. The coefficient B_0 (and, thus, A_0) must be null because the sphere is uncharged and, thus, in Eq. (16) the $1/r$ term must vanish. So, one gets

$$\phi^{(1)}_{\mathrm{ext}}(r,\theta)=\frac{a}{r}\sum_{n=1}^{\infty}A_n\left(\frac{a^2}{r}\right)^n P_n(\cos\theta). \tag{17}$$

Recalling again that A_n does not depend on a, we see from Eqs. (5) and (17) that

$$\phi^{(1)}_{\mathrm{ext}}(r,\theta)=\frac{a}{r}\phi^{(1)}_{\mathrm{int}}\left(\frac{a^2}{r},\theta\right). \tag{18}$$

(Since $\phi^{(1)}_{\mathrm{int}}$ is defined for $r\leqslant a$, the previous expression defines $\phi^{(1)}_{\mathrm{ext}}$ for $a^2/r\leqslant a$, i.e., for all $r\geqslant a$.)

The charge density on the surface can be computed as the discontinuity of the normal electric field, $\sigma(\theta)=\epsilon_0$

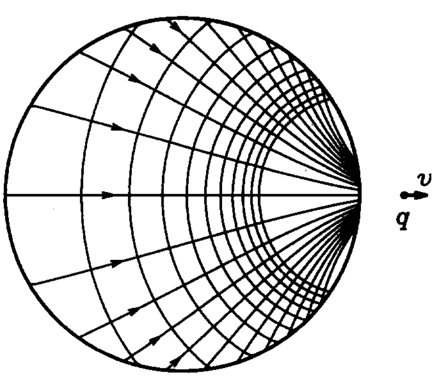

Fig. 2. First-order current lines and equipotential surfaces inside the sphere; equipotential surfaces close to the surface are not displayed.

$[E_{r\text{ext}}(a,\theta)-E_{r\text{int}}(a,\theta)]$ by using Eq. (18). At order zero, it is given by Eq. (1) while at first order one obtains, after an easy calculation

$$\sigma^{(1)}(\theta)=\epsilon_0(E^{(1)}_{r\text{ext}}(a,\theta)-E^{(1)}_{r\text{int}}(a,\theta))$$
$$=\epsilon_0\left[\frac{1}{a}\phi^{(1)}_{\text{int}}(a,\theta)-2E^{(1)}_{r\text{int}}(a,\theta)\right]. \qquad (19)$$

The time derivative of this first-order density gives a second-order contribution to the radial current density on the surface. This could be used to compute second-order fields, but as we shall discuss in Sec. VI one can expect that they are absolutely negligible in all practical cases.

Current lines (and thus electric field lines) and equipotential surfaces inside the sphere are drawn in Fig. 2, where the number of current lines is proportional to the current intensity. The difference of potential between adjacent equipotential surfaces is constant but surfaces closer to the charge are not displayed.

IV. FIRST-ORDER MAGNETIC FIELDS

As a consequence of the axial symmetry the magnetic field has only a nonvanishing component, $B=B_\phi(r,\theta)$, which does not depend on ϕ. It can be computed by using the first-order Ampère–Maxwell law

$$\oint_C \mathbf{B}^{(1)}\cdot d\mathbf{r}=\int_S\left[\mu_0\mathbf{j}^{(1)}+\frac{v}{c^2}\frac{\partial \mathbf{E}^{(0)}}{\partial x}\right]\cdot d\mathbf{S}. \qquad (20)$$

Using as the integration surface S the spherical cap of Fig. 3, which is concentric with the sphere and whose border C is a circumference defined by constant values or r and θ, one gets

$$B^{(1)}=\frac{r}{\sin\theta}\int_0^\theta \sin\alpha\left[\mu_0\, j_r^{(1)}(r,\alpha)+\frac{v}{c^2}\frac{\partial E_r^{(0)}}{\partial x}(r,\alpha)\right]d\alpha. \qquad (21)$$

Inside the sphere $E^{(0)}_{r\text{int}}=0$ and, so, there is no displacement current. The flux of the ohmic current $j_r^{(1)}=E^{(1)}_{r\text{int}}/\eta$ can be easily computed from Eq. (11) to get

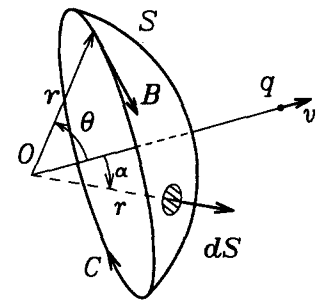

Fig. 3. Path and surface of integration for integrals in Eqs. (20) and (21).

$$B^{(1)}_{\text{int}}(r,\theta)=\frac{\mu_0 q v}{4\pi x^2\sin\theta}\left[\cos\theta-\frac{x^3\cos\theta-3x^2r+3xr^2\cos\theta-r^3}{(x^2-2xr\cos\theta+r^2)^{3/2}}\right]. \qquad (22)$$

Outside the sphere, on the contrary, the only contribution comes from the displacement current term and the Ampère–Maxwell law can be written in the form

$$2\pi r\sin\theta B^{(1)}_{\text{ext}}(r,\theta)=\frac{v}{c^2}\frac{\partial}{\partial x}(\Phi+\Phi'+\Phi''), \qquad (23)$$

where Φ, Φ', and Φ'' are, respectively, the flux through S of the electrostatic fields of q, and the two image charges q' and q''. The flux corresponding to a charge q_i located at a distance d_i from the center of the sphere along the segment joining the latter with q can be computed using S or any other surface having the same border. (In fact, the disk inside C or a spherical cap with center at the charge are more convenient to perform this calculation.) The result is

$$\Phi_i=\frac{q_i}{2\epsilon_0}\left[1-\frac{r\cos\theta-d_i}{\sqrt{d_i^2-2d_ir\cos\theta+r^2}}\right]. \qquad (24)$$

Taking $q_1=q$, $d_1=x$, $q_2=q'=-qa/x$, $d_2=a^2/x$, $q_3=q''=qa/x$, and $d_3=0$ in Eq. (24) and using Eq. (23) one finally gets

$$B^{(1)}_{\text{ext}}(r,\theta)=\frac{\mu_0 qva}{4\pi x^2 r\sin\theta}\left[\cos\theta+\frac{x^2r^2\sin^2\theta}{a(x^2-2xr\cos\theta+r^2)^{3/2}}+\frac{x^2a^2r^2\cos^2\theta-xr(x^2r^2+3a^4)\cos\theta+a^2(2x^2r^2+a^4)}{(x^2r^2-2xa^2r\cos\theta+a^4)^{3/2}}\right]. \qquad (25)$$

It is easy to check from Eqs. (22) and (25) that, in agreement with the assumption of no surface currents, the magnetic field is continuous across the sphere surface $r=a$. The result in Eq. (25) was obtained by Furry[5] in the case of a conductor in the form of a hollow spherical surface. Both results agree because they are due to the same real and image charges, but this is the only result that can be compared with those in Furry's work. For instance, the inside magnetic fields will be completely different because in our case there are currents in the bulk of the conductor.

V. POWER DISSIPATED BY THE JOULE EFFECT

Due to the Joule effect, inside the conducting sphere the following power is dissipated:

$$\frac{dW}{dt}=\int \mathbf{j}\cdot\mathbf{E}\,dV=\int\frac{E^2}{\eta}dV. \qquad (26)$$

The lowest order of this expression is the second one

$$\left(\frac{dW}{dt}\right)^{(2)}=\frac{2\pi}{\eta}\int_0^a\int_0^\pi(E^{(1)^2}_{r\text{int}}+E^{(1)^2}_{\theta\text{int}})r^2\sin\theta\,dr\,d\theta. \qquad (27)$$

By changing variables from (r,θ) to (r,z) with $z=\sqrt{x^2-2xr\cos\theta+r^2}$, the integrand in Eq. (27) is a rather lengthy but rational function. So, its integral can be performed by hand or, better, by using any computer algebra system. The result reads

$$\left(\frac{dW}{dt}\right)^{(2)}=\frac{q^2v^2\eta}{4\pi}\frac{a}{x^4}\left[\frac{a^2(11x^4-11x^2a^2+4a^4)}{(x^2-a^2)^3}-\ln\left(1-\frac{a^2}{x^2}\right)\right]. \quad (28)$$

Integration of this power over a finite time interval gives rise to a first-order energy loss. To have the charge q moving with constant velocity, an external force of value $-qE_{r\text{ext}}(x,0)$ must be applied on it. The first-order contribution of the power of this external force is stored in the electrostatic field in the form of potential energy while the second-order term,

$$-qE^{(1)}_{r\text{ext}}(x,0)v=qv\frac{\partial\phi^{(1)}_{\text{ext}}}{\partial r}(x,0), \quad (29)$$

supplies the energy dissipated inside the conductor by the Joule effect. In fact, one can easily check from Eqs. (13) and (18) that this second-order external power (29) is equal to Eq. (28).

If the charge is very close to the conductor the sphere surface appears as a plane. If d is the distance to the surface, putting $x=a+d$ in Eq. (28) and taking the limit when $d/a\to 0$, one gets

$$\left(\frac{dW}{dt}\right)^{(2)}=\frac{q^2v^2\eta}{8\pi d^3}, \quad (30)$$

in agreement with the result given in Ref. 1.

When the charge is far from the sphere the interior electric field is almost uniform and parallel to the charge velocity and when $a/x\to 0$, has the limit value $E^{(1)}_{r\text{int}}=E^{(1)}\cos\theta$ and $E^{(1)}_{\theta\text{int}}=-E^{(1)}\sin\theta$ where

$$E^{(1)}=\frac{3qv\eta}{2\pi x^3}, \quad (31)$$

and the dissipated power in this limit is

$$\left(\frac{dW}{dt}\right)^{(2)}=\frac{3q^2v^2\eta a^3}{\pi x^6}=\frac{E^{(1)^2}V}{\eta}, \quad (32)$$

V being the volume of the sphere.

VI. FINAL COMMENTS

To estimate the convergence speed of the expansion in powers of v/c, we can compare the induced dipole moments to orders 0 and 1. The charge distribution in Eq. (1) has a dipole moment of value

$$p^{(0)}=-\frac{qa^3}{x^2}, \quad (33)$$

while the first-order dipole moment obtained from Eq. (19) is

$$p^{(1)}=-\frac{6qv\eta\epsilon_0a^3}{x^3}. \quad (34)$$

Their ratio is $6\tau v/c$, being $\tau=\eta\epsilon_0c/x$. Since we assume $v/c\ll 1$, we expect that the dimensionless parameter τ is at most of the order of unity. For metallic conductors ($\eta\sim 10^{-8}\ \Omega$ m) this is true for distances $x\sim 10^{-10}$ m or greater. It turns out that we can expect that terms of order higher than the first will be completely negligible in all practical situations.

It is remarkable that first-order electric and magnetic fields inside the sphere are independent of the radius a, which is not the case for the exterior fields. On the other hand, interior and exterior first-order electric fields depend linearly on the resistivity η while magnetic fields are independent of it.

ACKNOWLEDGMENTS

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[1]T. W. Darling, F. Rossi, G. I. Opat, and G. F. Moorhead, "The fall of charged particles under gravity: A study of experimental problems," Rev. Mod. Phys. **64**, 237–257 (1992).
[2]T. H. Boyer, "Classical electromagnetic interaction of a charged particle with a constant-current solenoid," Phys. Rev. D **8**, 1667–1679 (1973); "Classical electromagnetic deflections and lag effects associated with quantum interference pattern shifts: Considerations related to the Aharonov–Bohm effect," *ibid.* **8**, 1679–1693 (1973).
[3]C. S. MacLatchy and H. A. Chipman, "A dynamic method of measuring the charge induced on a conductor," Am. J. Phys. **58**, 811–816 (1990).
[4]T. H. Boyer, "Penetration of the electric and magnetic velocity fields of a nonrelativistic point charge into a conducting plane," Phys. Rev. A **9**, 68–82 (1974).
[5]W. H. Furry, "Shielding of the magnetic field of a slowly moving point charge by a conducting surface," Am. J. Phys. **42**, 649–667 (1974).
[6]D. S. Jones, "The penetration into conductors of magnetic fields from moving charges," J. Phys. A **8**, 742–750 (1975).
[7]W. K. H. Panofsky and M. Phillips, *Classical Electricity and Magnetism*, 2nd ed. (Addison-Wesley, Reading, MA, 1962), p. 51.
[8]See. Ref. 7, p. 86.

THE DANGERS OF MARRYING A SCIENTIST

Men or women who go to the extreme length of marrying scientists should be clearly aware beforehand, instead of learning the hard way later, that their spouses are in the grip of a powerful obsession that is likely to take the first place in their lives outside the home, and probably inside too; there may not then be many romps on the floor with the children ...

P. B. Medawar, *Advice to a Young Scientist* (Harper & Row, New York, 1979), p. 22.