

# The atomic hypothesis: physical consequences

**Martín Rivas**

Theoretical Physics Department, The University of the Basque Country, Apdo. 644,  
48080 Bilbao, Spain

E-mail: [martin.rivas@ehu.es](mailto:martin.rivas@ehu.es)

Received 8 October 2007, in final form 21 January 2008

Published 15 July 2008

Online at [stacks.iop.org/JPhysA/41/304022](http://stacks.iop.org/JPhysA/41/304022)

## Abstract

The hypothesis that matter is made of some ultimate and indivisible objects, together with the restricted relativity principle, establishes a constraint on the kind of variables we are allowed to use for the variational description of elementary particles. We consider that the atomic hypothesis not only states the indivisibility of elementary particles, but also that these ultimate objects, if not annihilated, cannot be modified by any interaction so that all allowed states of an elementary particle are only kinematical modifications of any one of them. Therefore, an elementary particle cannot have excited states. In this way, the kinematical group of spacetime symmetries not only defines the symmetries of the system, but also the variables in terms of which the mathematical description of the elementary particles can be expressed in either the classical or the quantum mechanical description. When considering the interaction of two Dirac particles, the atomic hypothesis restricts the interaction Lagrangian to a kind of minimal coupling interaction.

PACS numbers: 11.30.Ly, 11.10.Ef, 11.15.Kc

(Some figures in this article are in colour only in the electronic version)

## 1. Introduction

Feynman, in the first chapter of his *Lectures on Physics* [1], states that ‘*If, in some cataclysm, all of scientific knowledge were to be destroyed, and only one sentence passed on to the next generations of creatures, what statement would contain the most information in the fewest words? I believe it is the atomic hypothesis (or the atomic fact or whatever you wish to call it) that all things are made of atoms—little particles that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one another.*’

If the atomic hypothesis is such an important principle, physics has to take advantage of this fact, and, properly formulated, should be included as a preliminary fundamental principle of elementary particle physics. The aim of this contribution is to show that the atomic

hypothesis not only states the indivisibility of elementary particles, but also that these ultimate objects, if not annihilated, cannot be modified so that all states of an elementary particle are only kinematical modifications of any one of them and, therefore, no excited states are allowed.

This paper is organized as follows. In the following section we give elementary geometrical arguments leading to the plausible conclusion that the most general motion of the centre-of-charge of a classical elementary spinning particle is a helical motion at the speed of light, so that the location of the charge satisfies, in general, a fourth-order differential equation, which is the most general differential equation satisfied by a point in three-dimensional space.

In the section 3 we analyse three fundamental principles, namely the restricted relativity principle, the atomic principle and the variational principle, which allow us to obtain a completely general formalism for describing, at the classical level, elementary spinning particles [2]. The quantization of this formalism is obtained by replacing the variational principle by the uncertainty principle in the form postulated by Feynman, i.e. in terms of the path integral approach.

In section 4 we summarize how the above fundamental principles produce a general kinematical formalism for describing elementary particles. Dirac's equation is obtained when quantizing precisely the classical system whose charge is moving along a helix, at the speed of light, as suggested by the above elementary arguments. The main features of a classical Dirac particle are outlined in section 5.

The importance of the atomic principle is stressed in section 6, when analysing the interaction of two Dirac particles. The atomic principle restricts the dependence of the interaction Lagrangian to the positions and velocities of both particles, but not to the accelerations and angular velocities. It is suggesting a kind of minimal coupling interaction between the currents of both particles. Finally, section 7 is devoted to some predictions of the kinematical formalism. It is shown that there is a difference in chirality between matter and antimatter, at the classical level. Matter is left handed while antimatter is right handed. It is also predicted that particles and antiparticles must necessarily have the same relative orientation between the spin and magnetic moment. The analysis of a very close electron–electron interaction shows that, if certain boundary conditions are fulfilled, two electrons with their spins parallel can form, from a classical point of view, a metastable bound state. We finish with some final conclusions.

## 2. Helical motion of the charge of an elementary spinning particle

As is well known in differential geometry, a continuous and differentiable curve in three-dimensional space,  $\mathbf{r}(s)$ , has associated three orthogonal unit vectors,  $\mathbf{t}$ ,  $\mathbf{n}$  and  $\mathbf{b}$ , called respectively the tangent, normal and binormal. If using the arc length  $s$  as the curve parameter, they satisfy the Frenet–Serret equations

$$\dot{\mathbf{t}} = \kappa \mathbf{n}, \quad \dot{\mathbf{n}} = -\kappa \mathbf{t} + \tau \mathbf{b}, \quad \dot{\mathbf{b}} = -\tau \mathbf{n},$$

where the overdot means  $\dot{\phantom{x}} \equiv d/ds$ . The knowledge of the curvature  $\kappa(s)$  and torsion  $\tau(s)$ , together the boundary values  $\mathbf{r}(0)$ ,  $\mathbf{t}(0)$ ,  $\mathbf{n}(0)$  and  $\mathbf{b}(0)$ , completely determine the curve, because the above equations are integrable. If we call  $\mathbf{r}^{(k)}(s) \equiv d^k \mathbf{r}/ds^k$ , and in particular  $\dot{\mathbf{r}} \equiv \mathbf{r}^{(1)} = \mathbf{t}$ , and eliminate the three unit vectors among the successive derivatives  $\mathbf{r}^{(k)}$ ,  $k \geq 1$ , one obtains that the most general differential equation satisfied by the point  $\mathbf{r}$ , is the fourth-order differential system

$$\mathbf{r}^{(4)} - \frac{2\dot{\kappa}\tau + \dot{\tau}\kappa}{\kappa\tau} \mathbf{r}^{(3)} + \left( \kappa^2 + \tau^2 + \frac{\dot{\kappa}\dot{\tau} - \tau\dot{\kappa}}{\kappa\tau} + \frac{2\dot{\kappa}^2}{\kappa^2} \right) \mathbf{r}^{(2)} + \frac{\kappa}{\tau} (\dot{\kappa}\tau - \dot{\tau}\kappa) \mathbf{r}^{(1)} = 0. \quad (1)$$

Let us consider that an elementary particle is a localized mechanical system. By localized we mean that, at least, is described by the evolution of a single point  $r$ . This point could be the centre-of-mass, but in order to determine the external forces to obtain the centre-of-mass evolution, we also need to know the location of the centre-of-charge to compute the actions of the external fields. Let us assume that the elementary particle is charged. Its electric structure can be reduced to the location of the centre-of-charge  $r$  and the subsequent multipoles located at this point. If its electric field is spherically symmetric we are reduced to know the evolution just of the centre-of-charge. We do not make the assumption that the centre-of-mass and the centre-of-charge are necessarily the same point. As we shall see this is not true for spinning particles, where the centre-of-mass evolution is some average of the evolution of the centre-of-charge.

By the previous arguments, the centre-of-charge of an elementary particle will satisfy, in general, a fourth-order differential equation of the form (1) where  $\kappa(s)$  and  $\tau(s)$  will depend on the external interaction. Let us assume now that the motion of the particle is free. This means that we cannot distinguish one point of the evolution from another, so that the above equations (1) must be explicitly independent of the parameter  $s$ . The curvature and torsion are necessarily constants of the motion. Thus  $\dot{\kappa} = \dot{\tau} = 0$ , and, in the free case, these equations are reduced to

$$r^{(4)} + (\kappa^2 + \tau^2)r^{(2)} = \frac{d^2}{ds^2}(r^{(2)} + (\kappa^2 + \tau^2)r) = 0.$$

If the curvature and torsion are constant the curve is a helix, which can be factorized in terms of a central point

$$q = r + \frac{1}{\kappa^2 + \tau^2}r^{(2)}, \quad \frac{d^2q}{ds^2} = 0,$$

which is moving in a straight trajectory, while the point  $r$  satisfies

$$r^{(2)} + (\kappa^2 + \tau^2)(r - q) = 0,$$

an isotropic harmonic motion of frequency  $\omega = \sqrt{\kappa^2 + \tau^2}$ , around point  $q$ . The point  $q$  clearly represents the centre-of-mass position of a free particle. Going further, let us assume that the free evolution is analysed by some inertial observer. Then this observer cannot distinguish one instant from another, so that, the arc length  $ds = |u| dt$ , where  $u = dr/dt$  is the velocity of the point, must be also independent of the time  $t$ . The centre-of-charge of a free elementary particle is describing a helix at a constant velocity for any inertial observer.

If we make a nonrelativistic analysis, the relationship of the velocity measurements among two arbitrary inertial observers  $O$  and  $O'$ , is given by  $u' = Ru + v$ , where  $v$  is the constant velocity of  $O$  as measured by  $O'$  and the constant rotation matrix  $R$  is their relative orientation. Now,

$$u'^2 = u^2 + v^2 + 2v \cdot Ru.$$

If  $u'$  has to be also constant for observer  $O'$ , irrespective of  $v$  and of the rotation matrix  $R$ , this means that the vector  $u$  must be a constant vector. The centre-of-charge necessarily moves along a straight trajectory at a constant velocity, for every inertial observer, and the above general helix degenerates into a straight line and  $q = r$ . This is the usual description of the spinless or pointlike free elementary particle, whose centre-of-charge and centre-of-mass are represented by the same point.

However, in a relativistic analysis, there is one alternative not included in the nonrelativistic approach. The possibility that the charge of an elementary particle will be moving at the speed of light and, in that case,  $u = u' = c$ , for any inertial observer. This

means that the centre of the helix is always moving at a velocity  $|dq/dt| < c$ , and, as will be shown, it represents the centre-of-mass, this particle is a massive particle. In a variational description of this system the Lagrangian should depend up to the acceleration of the point  $r$  in order to obtain fourth-order differential equations. This dependence on the acceleration will give a contribution to the spin of the particle. The motion of the charge around the centre-of-mass produces the magnetic moment of the particle.

In summary, there are only two possibilities for a free motion of the charge of an elementary particle. One, the charge is moving along a straight line at any constant velocity, and the system has no magnetic moment. In the other, the particle has spin and magnetic moment, the centre-of-mass and centre-of-charge are different points and the charge moves along a helix at the speed of light. Because all known elementary particles, quarks and leptons, are spin 1/2 particles, we are left only with the last possibility. This is consistent with Dirac's theory of the electron, because the eigenvalues of the components of Dirac's velocity operator are  $\pm c$  and we can interpret the corresponding point as representing the centre-of-charge.

This last possibility is the description of the centre-of-charge of a relativistic spinning elementary particle obtained in the proposed general kinematical formalism [2], and which satisfies Dirac's equation when quantized.

In this formalism Dirac particles are localized and also orientable mechanical systems. By orientable we mean that we have to attach to the above point  $r$ , which represents the position of the charge, a local cartesian frame to describe its spatial orientation. The rotation of the frame will also contribute to the total spin of the particle. When quantizing the system, the spin 1/2 is coming from the presence of the orientation variables. Otherwise, if there are no orientation variables, no spin 1/2 structure is described when quantizing the system. The dependence of the Lagrangian on the acceleration is necessary for the particle to have magnetic moment and for the separation between the centre-of-mass and centre-of-charge.

### 3. Fundamental principles

The restricted relativity principle states that, in absence of gravitation, there exists a set of equivalent observers, historically called *inertial observers*, for whom the laws of physics must be the same. This statement is an empty statement if not complemented with the assumption that the way two equivalent observers relate the measurement of any physical magnitude depends only of how they relate the measurements of spacetime events. They are thus defined with respect to each other by a spacetime transformation. The set of these transformations for all observers form a group, the kinematical group, which must be defined as the fundamental mathematical object of the formalism. It is this *geometrization* of spacetime which establishes the mathematical framework of the relativity principle.

The atomic principle admits that matter cannot be divided indefinitely. After a finite number of steps in the division of a portion of matter we reach an ultimate object, an *elementary particle*. In this way all known matter is finally made of these atom-little particles. Then, what is the difference between an elementary particle and any other little system? We need to distinguish theoretically a true elementary particle from a bound system of elementary particles. Otherwise the atomic hypothesis will be also an empty statement. This requires a proper definition of an elementary particle. The idea is that an electron, if not annihilated with its antiparticle, always remains an electron in any process of interaction. It thus means that an elementary particle has no excited states and, if not destroyed, we can never modify its internal structure, so that all possible states are only kinematical modifications of any one of them. If the state of an elementary particle changes, it is always possible to find another inertial observer who describes the particle in the same state as in the previous instant.

The variational principle recognizes that the action of the evolution of any mechanical system between some initial and final states must be stationary. This completes the classical framework. For the quantum description we must substitute this last variational principle by the uncertainty principle, in the form proposed by Feynman: all paths of the evolution of any mechanical system between some initial and final states are equally probable. For each path a probability amplitude is defined, which is a complex number of the same magnitude but whose phase is the action of the system between the end points  $x_1$  and  $x_2$  along the corresponding path. Feynman's total probability amplitude  $K(x_1, x_2)$  is the sum, or path integral, of the probability amplitudes for all paths joining these points. If we call kinematical variables to these classical variables which define the initial and final states of the variational description, these variables become, after quantization, the arguments of the wavefunction. In this way, classical and quantum mechanics are described in terms of exactly the same set of classical variables and its dynamics in terms of initial and final kinematical states. We want to emphasize the importance of the identification of the kinematical variables, and the interest of rewriting the Lagrangian formalism in terms of these variables.

#### 4. The kinematical formalism

The definition of elementary particle implies that its states can be described by a finite set of variables. Let us represent by  $x_1$  the values of the fixed set of variables which define the initial variational state, and, similarly, by  $x_2$  the final values of these variables. If the system is elementary, the final state  $x_2$  is a kinematical modification of  $x_1$ , so that there will exist some kinematical group element  $g$  such that  $x_2 = gx_1$ , for any  $x_1$  and  $x_2$ . The kinematical variables, which define the initial and final states of the evolution in the variational description, are a finite set of variables which necessarily span a homogeneous space of the kinematical group. The manifold they span is larger than the configuration space and, in addition to the independent degrees of freedom, it also includes the derivatives of the degrees of freedom up to one order less than the highest order they have in the Lagrangian. The Lagrangian for describing these systems will be thus dependent on these kinematical variables  $x$  and their next order time derivative. If the evolution is described in terms of some group invariant evolution parameter  $\tau$ , then, when writing the Lagrangian not in terms of the independent degrees of freedom but as a function of the kinematical variables and their  $\tau$ -derivatives,  $\dot{x}$ , it becomes a homogeneous function of first degree of the  $\tau$ -derivatives of all kinematical variables. This feature will allow us to make a theoretical analysis without postulating any particular Lagrangian.

The formalism is completely general and can accommodate to any kinematical group we consider as the spacetime symmetry group of the theory. But at the same time it is very restrictive, because once this kinematical group is fixed the kind of classical variables which define the initial and final states of an elementary particle in a variational approach, are restricted to belong to homogeneous spaces of the group. This kinematical group is the fundamental object of the formalism and, therefore, we call the formalism *kinematical*, to stress this fact.

All elementary systems described within this formalism have the feature that, when quantized, their Hilbert space of pure states carries a projective unitary irreducible representation of the kinematical group. It is through Feynman's path integral approach that both formalisms complement each other. For the Galilei and Poincaré groups, the most general homogeneous space is spanned by a set of ten variables, the same number and with the same geometrical interpretation as the group parameters,  $(t, r, u, \alpha)$ , interpreted as the time, position of the charge, velocity of the charge and orientation, respectively. In the relativistic case we have three maximal, disjoint, homogeneous spaces spanned by these

variables, according to the value of the velocity  $u < c$ ,  $u = c$  and  $u > c$ . The quantization of the manifold with  $u = c$ , produces Dirac's equation [2]. If  $x \equiv (t, \mathbf{r}, \mathbf{u}, \boldsymbol{\alpha})$  are the kinematical variables, then the Lagrangian will also depend on the acceleration and on the angular velocity. The dynamical equations for the point  $\mathbf{r}$  will be, in general, of fourth order.

A general spinning elementary particle is just a localized and orientable mechanical system. By localized we mean that to analyse its evolution in space we have just to describe the evolution of a single point  $\mathbf{r}$ , where the charge is located and in terms of which the possible interactions are determined. This point  $\mathbf{r}$  also represents the centre-of-mass of the system for spinless particles, while for spinning ones must necessarily be a different point than  $\mathbf{q}$ , the centre-of-mass, very well defined classically and where we can locate the mass of the particle. It is the motion of the charge around the centre-of-mass which gives rise to a classical interpretation of the *zitterbewegung* and also to the dipole structure of the particle. By orientable we mean that in addition to the description of the evolution of the point charge we also need to describe the change of orientation of the system  $\boldsymbol{\alpha}$ , by analysing the evolution of a local comoving and rotating frame attached to that point. An elementary spinning particle is thus described as we use to describe a rigid body but with some differences: we have not to talk about size or shape and the point does not represent the centre-of-mass but rather the centre-of-charge. It is allowed to satisfy a fourth-order differential equation and, for a Dirac particle, it moves at the speed of light.

## 5. A Dirac particle

This model of elementary spinning particle was already quantized through Feynman's path integral method [4] and shown to satisfy Dirac's equation. Therefore it corresponds to a classical spinning model of a spin 1/2 object when quantized. The classical expression which gives rise to Dirac's equation is

$$H = \mathbf{P} \cdot \mathbf{u} + \frac{1}{c^2} \mathbf{S} \cdot \left( \frac{d\mathbf{u}}{dt} \times \mathbf{u} \right),$$

where the energy  $H$  is expressed as the sum of two terms,  $\mathbf{P} \cdot \mathbf{u}$ , or translational energy and the other, which depends on the spin of the system, or rotational energy. This part can never vanish for any observer, while the first one is zero for the centre-of-mass observer. The spin comes from the dependence of the Lagrangian of both, the acceleration  $\dot{\mathbf{u}}$ , and the angular velocity  $\boldsymbol{\omega}$ , and if we define

$$\mathbf{U} = \frac{\partial L}{\partial \dot{\mathbf{u}}}, \quad \mathbf{W} = \frac{\partial L}{\partial \boldsymbol{\omega}},$$

it takes the form

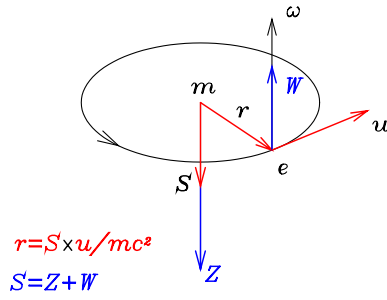
$$\mathbf{S} = \mathbf{u} \times \mathbf{U} + \mathbf{W} = \mathbf{Z} + \mathbf{W}.$$

The first part  $\mathbf{Z} = \mathbf{u} \times \mathbf{U}$ , or *zitterbewegung* part, is related to the separation between the centre-of-charge from the centre-of-mass and takes into account this relative orbital motion. It quantizes with integer values. The second part  $\mathbf{W}$  is the rotational part of the body frame and quantizes with both integer and half-integer values. The total angular momentum with respect to the origin of observer's frame is

$$\mathbf{J} = \mathbf{r} \times \mathbf{P} + \mathbf{S},$$

so that the spin  $\mathbf{S}$  is the angular momentum of the system with respect to the centre-of-charge  $\mathbf{r}$ , and not with respect to the centre-of-mass  $\mathbf{q}$ . This is the reason why for a free particle it is not a conserved quantity, but it satisfies the dynamical equation

$$\frac{d\mathbf{S}}{dt} = \mathbf{P} \times \mathbf{u}. \quad (2)$$



**Figure 1.** Motion of the charge of the electron in the centre-of-mass frame. The magnetic moment of the particle is produced by the motion of the charge. The total spin  $S$  is half the value of the zitterbewegung part  $Z$  when quantizing the system, so that when expressing the magnetic moment in terms of the total spin we get a  $g = 2$  gyromagnetic ratio [5]. The body frame attached to the end of point  $r$ , which could be the Frenet–Serret triad, rotates with angular velocity  $\omega$ , has not been depicted.

This is exactly the same dynamical equation satisfied by Dirac’s spin operator in the quantum case. If at point  $r$  there is defined some external force  $F$ , the total angular momentum is no longer conserved and thus

$$\frac{dJ}{dt} = r \times F = r \times \frac{dP}{dt} + u \times P + \frac{dS}{dt}$$

and because  $dP/dt = F$ , Dirac’s spin  $S$  also satisfies the dynamical equation (2) for an interacting particle. This has to be taken into account when comparing the analysis of this spin with other approaches, for instance, with Bargmann–Michel–Telegdi spin observable [3], which clearly represents the angular momentum with respect to the centre-of-mass of the system.

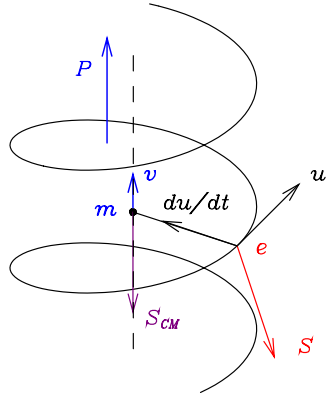
When expressed the spin and the centre-of-mass position in terms of the kinematical variables and their derivatives, they take, respectively, the form

$$S = \left( \frac{H - u \cdot P}{(du/dt)^2} \right) \frac{du}{dt} \times u, \quad q = r + \frac{c^2}{H} \left( \frac{H - u \cdot P}{(du/dt)^2} \right) \frac{du}{dt}.$$

Dirac’s spin is always orthogonal to the osculator plane of the trajectory of the charge  $r$ , in the direction opposite to the binormal for a positive energy particle, and the acceleration is pointing from  $r$  to the centre-of-mass, like in a helix. It is shown that the dynamical equation of point  $r$  for the free particle and in the centre-of-mass frame is given by

$$r = \frac{1}{mc^2} S \times u, \tag{3}$$

and where the spin vector  $S$  is constant in this frame, as depicted in figure 1. The radius of the zitterbewegung motion is  $R = S/mc$ , and the angular velocity  $\omega = mc^2/S$ . When considered in the centre-of-mass frame and all translational degrees of freedom are suppressed, it is a system of three degrees of freedom; two are the  $x$  and  $y$  components of the position of the charge on the zitterbewegung plane and the third is the phase of the rotation of the body frame. This phase is the same as the phase of the orbital motion and because the velocity  $u = c$  is constant, we are just left with a single and independent degree of freedom, for instance, the  $x$  coordinate. The Dirac particle, in the centre-of-mass frame, is a one-dimensional harmonic oscillator of frequency  $\omega$ .



**Figure 2.** Precession of Dirac’s spin along the linear momentum  $P$ . The transversal motion takes a time  $\gamma(v)$  longer to complete a turn. The three vectors  $u$ ,  $du/dt$  and  $-S$ , properly normalized, form the Frenet–Serret triad of the motion of the charge. The spin with respect to the centre-of-mass  $S_{CM}$ , is a constant of the motion for the free particle.

We can allow the system to have excited states, but the atomic principle suggests that an elementary particle cannot have excited states and that the only allowed state in the centre-of-mass frame, corresponds to the ground state. Its quantized ground energy  $\hbar\omega/2$  is identified with the particle rest frame energy  $mc^2$ . In this way, because  $\omega = mc^2/S$ , the classical spin  $S$  takes the value  $\hbar/2$ , when quantized. If the state of the particle, in the centre-of-mass frame, was some other excited state the value of the classical parameter  $S$  will be different than  $\hbar/2$ , and thus contradictory with the condition that this system satisfies Dirac’s equation. Therefore, the atomic hypothesis interpreted in the sense that the system has no excited states produces the same result for the quantized spin than Feynman’s quantization.

When seen from an arbitrary observer (see figure 2), the motion of the charge is a helix, so that according to (2) Dirac’s spin precess around the direction of the conserved linear momentum  $P$ . For a free particle, the centre-of-mass spin

$$S_{CM} = S + (r - q) \times P,$$

is a conserved quantity. The centre-of-mass velocity is  $v = dq/dt$ , and the linear momentum is written as usual as  $P = \gamma(v)mv$ , so that the transversal motion of the charge is at the velocity  $\sqrt{c^2 - v^2}$ . A moving electron takes a time  $\gamma(v)$  times longer than for an electron at rest to complete a turn, as a result of the time dilation measurement.

In a recent work [6] we have shown that the spacetime symmetry group for a Dirac particle, can be enlarged to include also spacetime dilations and local rotations of the body frame. This group is  $\mathcal{W} \otimes SO(3)_L$ , where  $\mathcal{W}$  is the Weyl group, i.e. the Poincaré group  $\mathcal{P}$  enlarged with spacetime dilations and  $SO(3)_L$  is the group of local rotations of the body frame, which commutes with  $\mathcal{W}$ . Because the Weyl group has no central extensions [7], the Lagrangian for a free Dirac particle is also invariant under this enlarged group.

If we consider this new group as the kinematical group of the theory, then the kinematical variables of a Dirac particle are reduced to time  $t$ , position of a point  $r$ , where the charge of the particle is located, its velocity  $u$  with the constraint  $u = c$ , the orientation  $\alpha$  which can be interpreted as the orientation of a local frame with origin at point  $r$  and characterized by three parameters of a suitable parameterization of the rotation group and, finally, a dimensionless scale  $\beta$  of the internal motion of the charge around the centre-of-mass. If the particle has spin  $S \neq 0$  and mass  $m \neq 0$ , then a length scale factor  $R = S/mc$  and a time scale factor



$T = S/mc^2$  can be defined, such that all kinematical variables for the variational description can be taken dimensionless. It is this argument which justifies the enlargement of the spacetime symmetry group, to include spacetime dilations which preserve the speed of light.

The Casimir operators of the enlarged group are the absolute value of the spin  $S$ , which is the Casimir operator of the Weyl group  $\mathcal{W}$ , and the absolute value  $I$  of the spin projection operator on the body frame of the rotational part of the spin

$$I_i = e_i \cdot \mathbf{W}$$

which corresponds to the Casimir operator of the  $SO(3)_L$  part. Here  $e_i, i = 1, 2, 3$  represent the three unit vectors of the local frame attached to the point  $r$ .

A Dirac particle, with the enlarged group  $\mathcal{W} \otimes SO(3)_L$  as its kinematical group, has as intrinsic properties the spin  $S$  and the spin projection  $I$  which take both the eigenvalue  $1/2$  when quantized [6]. By this reason, the four components of Dirac's spinor can be classified according to the  $\pm 1/2$  eigenvalues of the  $S_3$  and  $I_3$  components of these spin operators.

## 6. Physical consequences

Another important aspect of the atomic principle appears when we analyse the interaction between two Dirac particles [8]. The general structure of the free Lagrangian is

$$L_0 = T\dot{t} + \mathbf{R} \cdot \dot{\mathbf{r}} + \mathbf{U} \cdot \dot{\mathbf{u}} + \mathbf{W} \cdot \boldsymbol{\omega} + B\dot{\beta}, \quad (4)$$

because the Lagrangian is a homogeneous function of first degree in terms of the derivatives of the kinematical variables. Here,  $T = \partial L_0 / \partial \dot{t}$ ,  $\mathbf{R} = \partial L_0 / \partial \dot{\mathbf{r}}$ ,  $\mathbf{U} = \partial L_0 / \partial \dot{\mathbf{u}}$ ,  $\mathbf{W} = \partial L_0 / \partial \boldsymbol{\omega}$  and  $B = \partial L_0 / \partial \dot{\beta}$ . When we consider a compound system of two Dirac particles, the general Lagrangian will have the form  $L = L_1 + L_2 + L_I$ , in terms of the free Lagrangians  $L_1$  and  $L_2$  for each particle and an interaction Lagrangian  $L_I$ . The free Lagrangian for each particle, will have the general form (4) in terms of the corresponding kinematical variables of each particle. The interaction Lagrangian will be, in general, a homogeneous function of first degree in terms of the derivatives of all kinematical variables of both particles. But if we assume the atomic principle, the internal structure of each particle cannot be modified. This means that the spin  $S$  and the spin projection on the body frame  $I$  for each particle have to be obtained only from the corresponding free Lagrangian. This forbids the dependence of the interaction Lagrangian on the acceleration and angular velocity of the particles. A final invariance under spacetime dilations to obtain a Lagrangian invariant under the new kinematical group  $\mathcal{W} \otimes SO(3)_L$ , gives rise to the interaction Lagrangian

$$L_I = g \sqrt{\frac{c^2 \dot{t}_1 \dot{t}_2 - \dot{\mathbf{r}}_1 \cdot \dot{\mathbf{r}}_2}{(r_2 - r_1)^2 - c^2 (t_2 - t_1)^2}}, \quad (5)$$

where  $g$  is a coupling constant and the subindexes refer to the corresponding particles. This Lagrangian is clearly invariant under the interchange  $1 \longleftrightarrow 2$  of both particles. When making a synchronous description for any arbitrary observer, it becomes

$$L_I = g \sqrt{\frac{c^2 - \mathbf{u}_1 \cdot \mathbf{u}_2}{(r_2 - r_1)^2}} = g \frac{\sqrt{c^2 - \mathbf{u}_1 \cdot \mathbf{u}_2}}{r}, \quad (6)$$

where  $r = |\mathbf{r}_1 - \mathbf{r}_2|$  is the instantaneous separation between the corresponding charges. When the spin of both particles is suppressed, by taking in the low energy limit, the average values of the velocities of both charges will vanish, and the Lagrangian becomes the instantaneous Coulomb Lagrangian between two point charges, thus suggesting that  $gc = \pm e^2$ . It is with

the use of this interaction Lagrangian that the formation of bound states will be analysed in the following section.

In quantum electrodynamics the interaction Lagrangian between Dirac particles is obtained through the local gauge invariance prescription for the Dirac field. This requirement predicts the existence of a massless spin 1 field so that the interaction between Dirac particles is mediated through the gauge photon field in the form  $j^\mu A_\mu$ , where the particle current  $j^\mu = e\bar{\psi}\gamma^\mu\psi$  is coupled to the electromagnetic field  $A_\mu$ . In classical physics we have no means to describe, in a system of a finite number of degrees of freedom, the possibility of changing the number of particles and how intermediate particles could be created. We express the interaction only in terms of the classical variables associated with each particle. The atomic principle has been used here to restrict, among the possible interaction Lagrangians, those which do not modify the spin structure of any of the classical spinning particles.

To see if the classical Lagrangian (5) describes something equivalent to the quantum mechanical interaction Lagrangian of the Dirac field with the intermediate gauge field we have to prove that our classical Lagrangian can be rewritten, for instance, in the form of a coupling of each particle current with the retarded classical electromagnetic potential of the other, i.e.  $j_1^\mu A_{2\mu} + j_2^\mu A_{1\mu}$  or something alike. However, this Lagrangian describes an action at a distance interaction between particles in the form of a coupling of the particles four velocities  $\dot{x}_1^\mu \dot{x}_{2\mu}$  and in terms of their spacetime separation  $(x_1 - x_2)^2$ , but not in terms of the retarded spacetime positions so that, if the above decomposition could be achieved, the Lagrangian could be interpreted as the predictivization of the retarded electromagnetic interaction between the two particles. This task is, probably, cumbersome and out of the scope of the present section which tries to enhance the role of the atomic principle in restricting the allowed interactions. It poses an interesting research subject for future work. But, nevertheless, the present Lagrangian contains as a limit, when the spins of the particles are suppressed, the instantaneous Coulomb interaction between the two point charges, which is a nice and expected nonrelativistic and spinless limit.

If we succeed in showing this feature it would mean some relationship between the quantum local gauge invariance statement and the atomic principle because they lead, in the quantum and classical framework, respectively, to a similar interaction description. It is an interesting theoretical ansatz.

## 7. Predictions of the formalism

The formalism produces several predictions:

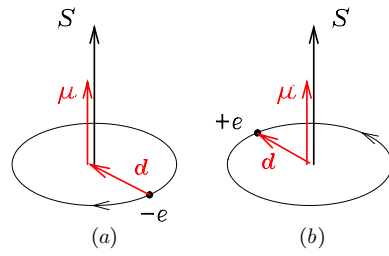
- Chirality. Matter is left handed and antimatter right handed.
- Particles and antiparticles have the same relative orientation between the spin and magnetic moment.
- A repulsive force between charges does not forbid the formation of bound states, provided the spins are parallel.

The conserved kinematical momentum, i.e. the constant of the motion associated with the invariance of the Lagrangian under Lorentz boosts, takes the form for a Dirac free particle

$$\mathbf{K} = H\mathbf{r}/c^2 - \mathbf{P}t - \mathbf{S} \times \mathbf{u}/c^2.$$

In the centre-of-mass frame  $\mathbf{P} = \mathbf{K} = 0$  and  $H = \pm mc^2$ , and Dirac's spin  $\mathbf{S}$  is a constant vector, so that the motion of the charge is given by

$$\mathbf{r} = \pm \frac{\mathbf{S} \times \mathbf{u}}{mc^2}, \quad (+\text{particle}), \quad (-\text{antiparticle}),$$



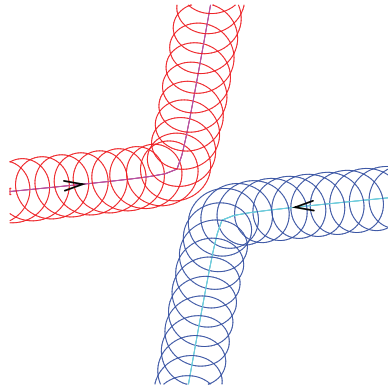
**Figure 3.** (a) Motion of the charge, showing the electric and magnetic dipole with respect to the centre-of-mass, for the positive energy particle. (b) The PCT transformed system corresponds to the antiparticle, with the same relative orientation between spin and magnetic moment. Once the spin direction is fixed as a positive direction, the motion of the charge for the particle (a) is clockwise, which corresponds to a negative or left-handed motion. Antimatter (b), corresponds to a right-handed motion. The instantaneous electric dipole with respect to the centre-of-mass is defined as  $d = \pm e(r - q)$ .

and these two motions are depicted, respectively, in figures 3(a) and (b), where we have chosen for the particle the charge  $-e$  and for the antiparticle  $+e$ . There is an arbitrariness in the selection of the charge of the particle, but the motion of the positive energy solution is clockwise, once the spin direction is fixed, while for the antiparticle we have a counterclockwise motion, although both motions produce the same spin. Then, the particle makes a negative trajectory in the zitterbewegung plane, thus representing a left-handed system. Antimatter moves according to a right-handed system.

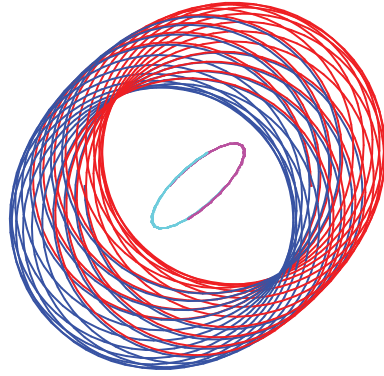
This produces that particle and antiparticle have the same magnetic dipole with the same relative orientation with respect to the spin. They also have an instantaneous electric dipole which rotates very fast around the spin direction, so that its time average is basically zero for low energy processes. The electron, as an average, can thus be considered as a point charge at rest and some magnetic dipole, located both at the centre-of-mass. But in very close electron–electron interaction or in high energy processes, both electric and magnetic dipoles have to be taken into account for describing the interaction, or, alternatively, the knowledge of the actual location of the corresponding charges.

As a matter of fact, the positronium (electron–positron bound system) has a ground state of spin 0 and magnetic moment 0. This means that the spins of both electron and positron are antiparallel to each other and the same thing happens to the corresponding magnetic moments. Therefore, for the electron and positron there would exist the same relative orientation between the spin and magnetic moment. The neutral pion  $\pi^0$  is a linear combination of the quark–antiquark bound systems  $u\bar{u}$ ,  $d\bar{d}$  and sometimes the pair  $s\bar{s}$  is also included. It is a system of 0 spin and 0 magnetic moment. Because each of the above quarks have different masses and charges, and thus different magnetic moments, the possibility is that each quark–antiquark pair is a system of 0 spin and 0 magnetic moment, and, therefore each quark and the corresponding antiquark must have the same relative orientation between the spin and magnetic moment.

This feature is opposite to what is usually assumed because for the electron it is taken that spin and magnetic moment are opposite to each other, while for the positron they are taken parallel. However, in my opinion, there is no clear experimental evidence in the literature of this fact and, therefore, experimentalists should check at least, for electrons and muons, whether they have the spin and magnetic moments parallel or antiparallel. One possibility is to analyse the motion of these particles in storage rings. If, as predicted, they have the same relative orientation, then when injecting in the same direction,  $e^+$  and  $e^-$  (or muons either),



**Figure 4.** Scattering of two electrons with their spins parallel. The intermediate trajectories, marked with an arrow, correspond to the evolution of the corresponding centres-of-mass.

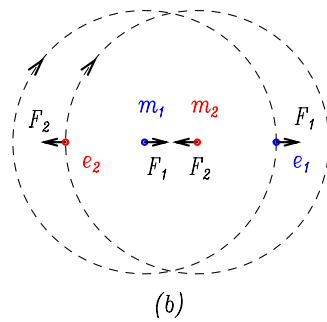


**Figure 5.** Bound motion of two electrons with the spins parallel, for an initial velocity of the centre-of-mass of each particle of  $v < 0.01c$ . The phases of the charges have to be, basically, opposite to each other to produce a metastable bound system, and the initial separation between the centres-of-mass is  $0.2 \times$  Compton's wavelength.

polarized in the up direction, their spins must precess in the opposite direction, because the magnetic field has to be reversed when we change from particles to antiparticles. The direction of precession will show whether they are parallel or antiparallel.

If we analyse the interaction of two Dirac particles we can use the mentioned interaction Lagrangian (6) which is invariant under the enlarged  $\mathcal{W} \otimes SO(3)_L$  group. When the two particles are far apart, the behaviour of the interaction becomes the instantaneous Coulomb interaction between the charges. In figure 4 we represent the scattering of two electrons with the spins parallel and where the trajectories of the corresponding centres-of-mass are also depicted. In this example the two particles approach each other to a separation greater than Compton's wavelength.

But if we locate very closely the two electrons, below Compton's wavelength, provided the phases of the charges in the internal motion are opposite to each other, and the velocities below  $0.01c$ , we can obtain metastable bound motions like the one depicted in figure 5. The mass of this spin 1 bound system is greater than  $2m_e$ , because the potential and kinetic energies are



**Figure 6.** Initial position of the charges of the two particles with the spins parallel and with a phase shift of  $180^\circ$ . The dotted lines represent the previsible evolution of each charge, in the same direction for both particles, which implies that the spins are parallel. The repulsive force between charges is also depicted at the corresponding centre-of-mass, thus producing an attractive force between the particles.

both positive. The solution of the corresponding quantum analysis, in particular the possible quantization of the binding energies, is not yet done. The analysis of this bound motion has been done in [8].

To justify how two particles of the same charge can attract each other, we have to solve a system of fourth-order differential equations for each particle or, alternatively, a system of second-order differential equations once the centres-of-mass of the particles are defined. For each centre-of-mass trajectory we need to know the external force acting on the corresponding particle, but this force is defined at the corresponding centre-of-charge, and as we see in figure 6 a repulsive force between the charges implies an attractive force between the centres-of-mass provided the phases of the charges are opposite to each other.

## 8. Summary and conclusions

In a schematic form we list briefly some general features and conclusions about the kinematical formalism, obtained by assuming the atomic hypothesis as a fundamental principle.

- An elementary particle is a system without excited states. If it is not destroyed, its internal structure can never be modified. All its possible states are kinematical modifications of any one of them.
- The most general trajectory of the charge of a free elementary spinning particle is a helix at the speed of light.
- The kinematical group supplies the symmetries and the variables for the variational description of an elementary particle, which necessarily span a homogeneous space of the group.
- These classical variables define the support manifold of the Hilbert space when quantizing the system.
- The kinematical formalism is complete in the sense that the quantization of the models produces all known one-particle wave equations.
- The spinning particles are localized and orientable systems.
- The centre-of-charge and centre-of-mass are necessarily different points.
- Elementary Dirac particles have a definite chirality. Matter is left handed and antimatter right handed.

- The spin has a twofold structure: one part is related to the orbital motion of the centre-of-charge and the other is related to the rotation of the particle.
- This twofold structure produces a kinematical interpretation of the gyromagnetic ratio.
- The magnetic moment is produced by the motion of the centre-of-charge around the centre-of-mass (zitterbewegung).
- A particle and its corresponding antiparticle have the same relative orientation between the spin and magnetic moment.
- The spacetime symmetry group of a Dirac particle is larger than the Poincaré group. It becomes, at least,  $\mathcal{W} \otimes SO(3)_L$ .
- It is the spin the only intrinsic property of a Dirac particle if considered under this kinematical group.
- Two equal charged particles can form, from the classical point of view, bound systems provided their spins are parallel and if their separation is below Compton's wavelength.

### Acknowledgments

I thank my colleague J M Aguirregabiria for the use of his excellent Dynamics Solver program [9] with which the numerical computations of the electron dynamics have been done. This work has been partially supported by Universidad del País Vasco/Euskal Herriko Unibertsitatea grant 9/UPV00172.310-14456/2002.

### References

- [1] Feynman R P, Leighton R B and Sands M 1968 *The Feynman Lectures on Physics* vol 1 (New York: Addison-Wesley) section 1–2
- [2] Rivas M 2001 *Kinematical Theory of Spinning Particles* (Dordrecht: Kluwer)  
See also Rivas M 2005 Lecture notes of the course *Kinematical Formalism of Elementary Spinning Particles*, (JINR, Dubna, 19–23 September 2005) Preprint [physics/0509131](http://physics/0509131) Some other works related to this formalism can be obtained through the webpage <http://tp.lc.ehu.es/martin.htm>
- [3] Bargmann V, Michel L and Telegdi V L 1959 *Phys. Rev. Lett.* **2** 435
- [4] Rivas M 1994 *J. Math. Phys.* **35** 3380
- [5] Rivas M, Aguirregabiria J M and Hernandez A 1999 *Phys. Lett. A* **257** 21
- [6] Rivas M 2006 *J. Phys. A: Math. Gen.* **39** 4291 (Preprint [hep-th/0511244](http://hep-th/0511244))
- [7] Boya L J, Cariñena J F and Santander M 1975 *J. Math. Phys.* **16** 1813
- [8] Rivas M 2007 *J. Phys. A: Math. Theor.* **40** 2541 (Preprint [hep-th/0702172](http://hep-th/0702172))
- [9] Aguirregabiria J M 2006 *Dynamics Solver* computer program for solving different kinds of dynamical systems, which is available from his author through the web site <http://tp.lc.ehu.es/jma.html> at the server of the Theoretical Physics Dept. of The University of the Basque Country, Bilbao, Spain